Appendix C

EXPOSURE TIME VERSUS PICTURE ELEMENT SIZE

Consider the time required to expose a pattern with a focused scanning electron beam. The electron beam with current density $J(A/cm^2)$ must strike a pixel for time τ (sec) to produce exposure Q (coulombs/cm^2) $= J\tau$. The beam current density $J = J_c(eV/kT)\alpha^2$ by Langmuir's law, where J_c, T, and V are cathode current density, temperature, and beam accelerating voltage, e and k are the electronic charge (1.6×10^{-19} coulombs) and Boltzmann's constant (1.38×10^{-23} J/°K), and α is the beam convergence angle.

By increasing α, the current density exposing the pattern increases, which is desirable. However, if α is increased too far, the beam spot diameter increases because of the spherical aberration of the focusing system. An optimum value of α occurs when the diameter of the disk of confusion due to spherical aberration, $d_s = 0.5\ C_s\alpha^3$ (C_s is the spherical aberration coefficient), is set equal to the gaussian spot diameter, $d_s = d_g = \ell_p/\sqrt{2}$. Using the normal approximation of adding spot diameters in quadrature, the total spot size then is $d = (d_s^2 + d_g^2)^{1/2} = \ell_p$, the pixel dimension. The optimum convergence angle is then

$$\alpha_{opt} \approx \left[\frac{\sqrt{2}\ \ell_p}{C_s}\right]^{1/3},$$

and the exposure in time τ is

$$Q = J\tau = J_c\,\frac{eV}{kT}\left[\frac{\sqrt{2}\ \ell_p}{C_s}\right]^{2/3}\tau = \frac{\beta\pi 2^{1/3}}{C_s^{\,2/3}}\,\ell_p^{\,2/3}\tau, \qquad (1)$$

where β is the electron optical brightness ($J_c eV/\pi kT$). Equation (1) gives the change density deposited in a spot of diameter ℓ_p in time τ. For resist exposure, this charge density must equal the resist sensitivity under the exposure conditions used.

To ensure that each pixel is correctly exposed, a minimum number of electrons must strike each pixel. Since electron emission is a random process, the actual number of electrons striking each pixel, n, will vary in a random manner about a mean value, $\bar{n}$. Adapting the signal-to-noise analysis found in Schwartz (1959) to the case of binary exposure of a resist, one can show straightforwardly that the probability of error for large values of the mean number of electrons/pixel $\bar{n}$ is $e^{-\bar{n}/8}/[(\pi/2)\bar{n}]^{1/2}$. This leads to the following table of probability of error of exposure:

$\bar{n}$	50	100	150	200
Probability of error	2.2×10^{-4}	3×10^{-7}	4.7×10^{-10}	7.8×10^{-13}

To be conservative, we choose $\bar{n} = 200$, which should mean that, on average, no pixels in a field of 10^{10} pixels are incorrectly exposed due to randomness, as long as each electron striking a pixel causes at least one exposure event in the resist. For a pixel of dimension ℓ_p, the minimum number of electrons striking it (= 200 here) to provide adequate probability of exposure is N_m, and the charge density is then $Q = N_m e/\ell_p^2$. Substituting into (1) gives

$$N_m e = \frac{\beta\pi 2^{1/3}}{C_s^{2/3}} \tau \ell_p^{8/3}. \qquad (2)$$

To determine how noise limits pixel dimension, arrange (2) so that normalized exposure time depends on pixel dimension; note that $2^{1/3}\pi \simeq 4$:

$$\left[\frac{4\beta}{N_m e C_s^{2/3}}\right] \tau = \ell_p^{-8/3}. \qquad (2a)$$

A corresponding equation for real resist exposure is

$$\left[\frac{4\beta}{N_m e C_s^{2/3}}\right] \tau_R = \frac{Q}{N_m e} \ell_p^{-2/3}. \qquad (1a)$$

Here the same normalization was chosen for τ to facilitate plotting (1a) and (2a) on the same figure of τ vs ℓ_p (see Fig. 2 of the text).